Single Point Binomials v. Interval Binomials

- Single Point Binomials
  - By hand
    - We use the probability distribution function!

$$p(x)=\binom{n}{x}p^x q^{n-x} \quad \& \quad x=0,1,2,\ldots,n$$

  - By calculator --> Binompdf
    - Components to know
      - n = number of trials or total
      - p = the probability of success
      - x = the point of interest

order → (n, p, x)

2nd → VARS → A: binompdf(

- Interval Binomials
  - By hand
    - We STILL use the probability distribution function BUT we use it for EVERY SINGLE POINT wanted in the interval!
  - By calculator --> Binomcdf
    - This calculator function calculates and includes every point in an interval that resembles $P(X \leq x)$ ← Given #
    - Components to know
      - n = number of trials or total
      - p = the probability of success
      - x = max. point in target interval

order → (n, p, x)

2nd → VARS → B: binomcdf(

- Other formulas for Binomial Random Variable
  - Mean

$$\mu = np$$

- Variance

$$\sigma^2 = npq$$

- Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Practice

1. A baker considers a cookie batch successful when 3 quarters of the batch of chocolate chip cookies has no less than 5 chocolate chips in each cookie. Each day, on average, the baker makes 8 batches of chocolate chip cookies (with each batch having 24 cookies) and only 6 are usually considered successes. Solve the questions below by hand.
   a. What is the probability of success and failure? (Hint: pay attention to the batches and not the number of cookies.)

$p = \frac{x}{n} = \frac{6}{8} = 0.75$ $\quad q = 1 - p = 1 - 0.75 = 0.25$

   b. Write the probability function.

$p(x) = \binom{8}{x}(0.75)^x(0.25)^{8-x} \quad x = 0, 1, \ldots, 8$

   c. What is the probability of 5 successes?

$P(X=5) = \binom{8}{5}(0.75)^5(0.25)^{8-5} = 0.21$

   d. What is the probability of more than 2 but at most 4 successes? $P(2 < x \leq 4)$ wants 3 & 4

$P(X=3) = \binom{8}{3}(0.75)^3(0.25)^{8-3} = 0.02$

$P(X=4) = \binom{8}{4}(0.75)^4(0.25)^{8-4} = 0.09$

$\Rightarrow P(2 < x \leq 4) = 0.11$

   e. What is the probability of less than 2 or of 7 successes? $P(X<2 \text{ or } X=7)$ wants 0, 1, & 7

$P(X=0) = \binom{8}{0}(0.75)^0(0.25)^{8-0} = 1.53 \times 10^{-5} = 0.0000153$

$P(X=1) = \binom{8}{1}(0.75)^1(0.25)^{8-1} = 3.66\times10^{-4} = 0.000366$

$P(X=7) = \binom{8}{7}(0.75)^7(0.25)^{8-7} = 0.27$ $P(X<2 \text{ or } X=7) = 0.27$

f. What is the mean, variance, and standard deviation?

$\mu = np = (8)(0.75) = 6$

$\sigma^2 = npq = (8)(0.75)(0.25) = 1.5$

$\sigma = \sqrt{\sigma^2} = \sqrt{1.5} = 1.22$

2. The Wingstop franchise expects that every order is completed after 12 minutes from receiving the order. On average, the Stephenville location meets expectations on 84% of all orders received each day. Solve the questions below by calculator if Wingstop in Stephenville had a total of 356 orders throughout the day.

a. What is the probability of success and failure?

$p = 84\% = 0.84$ $q = 1-0.84 = 0.16$

b. Write the probability function.

$p(x) = \binom{356}{x}(0.84)^x(0.16)^{356-x}$ $X = 0, 1, \ldots, 356$

c. What is the probability of 281 successes?

$P(X=281) = \text{binompdf}(356, 0.84, 281) = 0.002$

d. What is the probability of at most 300 successes?

$P(X\leq300) = \text{binomcdf}(356, 0.84, 300) = 0.58$

e. What is the probability of more than 250 successes but less than 320 successes?

$P(250<x<320) = P(x\leq319) - P(x\leq250)$

$P(x \leq 319) = \text{binomcdf}(356, 0.84, 319) = 0.9991$

$P(x \leq 250) = \text{binomcdf}(356, 0.84, 250) = 5.92 \times 10^{-11} \approx 0.0000$

$P(250 < x < 320) = 0.9991 - 0.0000 = 0.9991$

f. What is the probability of less than 108 or more than 280 successes?

$P(x < 108 \text{ or } x > 280) = P(x \leq 107) + [1 - P(x \leq 280)]$

$P(x \leq 107) = \text{binomcdf}(356, 0.84, 107) = 0$

$P(x \leq 280) = \text{binomcdf}(356, 0.84, 280) = 0.005$

$P(x < 108 \text{ or } x > 280) = 0 + [1 - 0.005] = 0.995$

g. What is the mean, variance, and standard deviation?

$\mu = np = (356)(0.84) = 299.04$

$\sigma^2 = npq = (356)(0.84)(0.16) = 47.85$

$\sigma = \sqrt{\sigma^2} = \sqrt{47.85} = 6.92$