Factorial and Combinations

- Factorial
  - Must be a positive integer
  - If the integer is 0, then the factorial of it is 1
  - By hand

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$$

  - Calculator Trick
    - Make sure to enter the number FIRST

→ MATH → PRB → 4:!

- Combinations
  - We focus on situations where $0 \leq n \leq 1$
  - By hand (aka formula)

$$nCx = \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

  - Calculator Trick
    - Make sure to enter the number (n) FIRST

n# → MATH → PRB → 3:nCr → x#

Practice

Solve the following questions! Do the odd numbers by hand, and the even numbers by calculator.

1. $4! = 4 \cdot 3 \cdot 2 \cdot 1$

$4! = 24$

2. $8!$

$8! = 40320$

3. $\frac{11!}{6!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!}$

$\frac{11!}{6!} = 55440$

4. $\frac{30!}{12!}$

$\frac{30!}{12!} = 5.54 \times 10^{23}$

5. ${}_5C_3 \rightarrow \binom{5}{3}$

$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} = \frac{5 \cdot 4}{2 \cdot 1}$

$\binom{5}{3} = 10$

6. ${}_{15}C_8 \rightarrow \binom{15}{8}$

$\binom{15}{8} = 6435$

Bernoulli v. Binomial

- Bernoulli
  - Has only 2 possible outcomes
    - We simply label them as success and failure
  - Variable wise, success is p and failure is q
  - p + q = 1
- Binomial
  - This is the summary of multiple Bernoulli trials
  - These types of questions always tell you:
    - The number of trials conduct
    - What the success or failure RATE
  - The function of a binomial distribution is formatted using...

$$p(x) = \binom{n}{x} p^x q^{n-x} \quad x = 0, 1, 2 \ldots n$$

    - ALWAYS include a list of possible x's (which starts at 0) when asked for a binomial distribution function!!

Practice

1. A student organization took a poll of what classifications each of the members were in college. They found that out of the 343 people, the distribution was as follows: 104 freshmen, 79 sophomores, 82 juniors, and 78 seniors. Knowing that success is defined as seniors, what is the probability of success and failure?

$p = \frac{x}{n} = \frac{78}{343} = 0.23$ $\quad q = 1 - p = 1 - 0.23 = 0.77$

2. The governor of a small town sees that a common trend of the town last year is that 18% of the population that shop at Walmart utilize food stamps. He surveyed a total of 100 people shopping at Walmart to see if this is true. If the success is those that do not use food stamps, what is the probability of success and failure?

$q = 18\% = 0.18$ $\quad p = 1 - q = 1 - 0.18 = 0.82$

3. A high school athlete running sprints is recording the time it takes to complete each sprint to track her progress. Currently, her personal best is 16.67 seconds. During a training session, she runs a total of 15 sprints at varying times. Out of the 15, she is able to get 7 sprints within 30 seconds from her personal best. (n, x)

   a. What is the probability of success and failure?

$p = \frac{7}{15} = 0.47$ $\quad q = 1 - 0.47 = 0.53$

   b. Write the probability function. (Hint: all functions must include the possible x-values)

$p(x) = \binom{15}{x}(0.47)^x(0.53)^{15-x}$ $\quad x: 0, 1, 2, \ldots, 15$

   c. What is the probability of 5 successes during her next session?

$p(5) = \binom{15}{5}(0.47)^5(0.53)^{15-5} = 0.12$ (3003)

   d. What is the probability of more than 3 but at least 5 successes?

$p(3 < x \leq 5)$ → wants 4 & 5

$p(4) = \binom{15}{4}(0.47)^4(0.53)^{15-4} = 0.06$ (1365)

$p(5) = 0.12$

$p(3 < x \leq 5) = 0.06 + 0.12 = 0.18$

e. What is the probability of 8 failures? → 7 successes

$$P(7) = \binom{15}{7}(0.47)^{7}(0.53)^{15-7} = 0.20$$

$\binom{15}{7} = 6435$

f. What is the probability of more than 13 successes or less than 2?

$P(X>13 \text{ or } X<2)$ → wants 14 & 15 + 0 & 1

$$P(14) = \binom{15}{14}(0.47)^{14}(0.53)^{15-14} = 2.04\times10^{-4} = 0.000204$$

$$P(15) = \binom{15}{15}(0.47)^{15}(0.53)^{15-15} = 1.21\times10^{-5} = 0.0000121$$

$$P(0) = \binom{15}{0}(0.47)^{0}(0.53)^{15-0} = 7.31\times10^{-5} = 0.0000731$$

$$P(1) = \binom{15}{1}(0.47)^{1}(0.53)^{15-1} = 9.73\times10^{-4} = 0.000973$$

$+$

$P(X>13 \text{ or } X<2) = 0.0012622$

-3

$P(X>13 \text{ or } X<2) = 1.26\times10^{-3}$