Chapter 2

Mean

1. A professor, who has two separate classes for the same subject, just finished grading everyone's grade on their first test of the semester. The professor wants to know the class average of each individual class, as well as the average of both classes. Using the data below, find the information that the professor wants.

C-1	76	98	95	83	97	89	72	65	92	85	57	63
C-2	82	67	84	93	70	74	99	78	86	60	54	
	1	2	3	4	5	6	7	8	9	10	11	12

Class 1 (C-1) Mean:

$$\bar{X}_1 = \frac{\sum x_1}{n} = \frac{76+98+\ldots+63}{12} = 81$$

Class 2 (C-2) Mean:

$$\bar{X}_2 = \frac{\sum x_2}{n} = \frac{82+67+\ldots+54}{12} = 77$$

Overall Mean:

$$\bar{X}_{1+2} = \frac{\bar{X}_1+\bar{X}_2}{2} = \frac{81+77}{2} = 79$$

2. Find the missing value using the data table below.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	11	29	15	17	8	22	40	?	8	2	33	18	27	43	20
Average: 21.4															

$$\bar{X} = \frac{\sum x}{n} \rightarrow \sum x = \bar{X} \cdot n = 21.4 \cdot 16 = 342.4$$

$$x(9) = \sum x - \left(\sum(1-8) + \sum(10-16)\right) = 342.4 - \left((4+11+\ldots+40)+(8+\ldots+20)\right) = 45.4$$

Median

Find the median of the data sets below. (Hint: remember to sort them first!)

A. 3, 2, -1, 6, 4, -3, 2, -12, 1, 1, -5, 4 → -12, -5, -3, -1, 1, 1, 2, 2, 3, 4, 4, 6

$n = 12$ $\frac{12}{2} = 6$th & 7th value

$\frac{1+2}{2} = 1.5$

B. 9, 6, -3, 18, 12, -9, 6, 3, 3 → -9, -3, 3, 3, 6, 6, 9, 12, 18

n = 9 exact middle

Mode

For the questions below, identify the mode(s).

A. 3, 8, 6, 3, 4, 2, 6, 4, 8, 2, 4

3: II 6: II 2: II
8: II 4: III

Mode is 4

B. 2, 9, 6, 9, 6, 3, 4, 2, 6, 3, 9, 2, 1, 2

2: IIII 6: III 4: I
9: III 3: II 1: I

Mode is 2

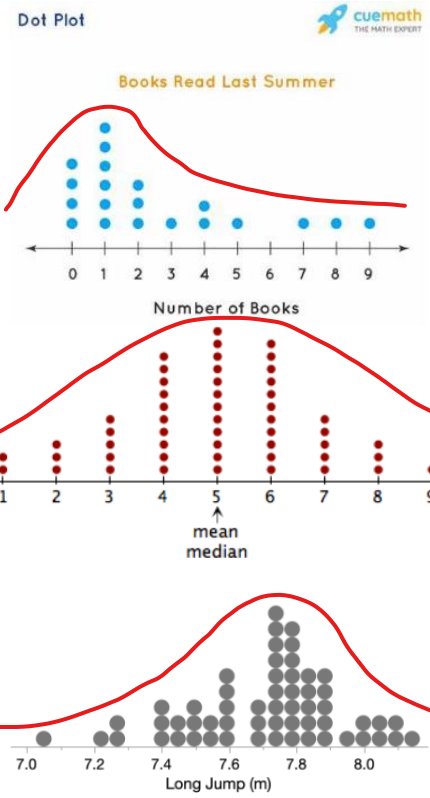
C. Look at the Bar Graph below.

Summary of Mean, Median, and Mode

Determine whether the graphs below are right-skewed, left-skewed, or symmetrical.

Regular: if bump of curve is on left, its right skewed & vice versa

Boxplots: model box with Q_1 & Q_3 as your hands & Q_2 as your body
→ left hand closer to body = right skewed
→ right hand closer to body = left skewed



Right skewed

Symmetrical

Left Skewed

Standard Deviation

1. Find the standard deviation of the data below by hand.

[89, 70, 56, 93, 84, 87, 80, 76]

$\Sigma x = 89+70+\ldots+76 = 635$

$\Sigma x^2 = 89^2+70^2+\ldots+76^2 = 51407$

$n = 8$

$$S = \sqrt{\frac{(\Sigma x^2) - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{(51407) - \frac{(635)^2}{8}}{8-1}} = 11.98$$

2. Find the standard deviation of the data below by calculator. → 1-Var Stat

[42, 17, 20, 89, 23, 56, 78, 34, 91, 65, 65, 12] → L1

$S_x = 28.52$

Five Number Summary and Modified Box Plots

A. Using the data below, find the five number summary.

[2, 9, 1, 4, 8, 7, 10, 8, 6, 1, 10, 8, 5, 3, 6, 8] → n=16 → $\frac{8+8}{2} = Q_2$

1, 1, 2, 3, 4, 5, 6, 6, 7, 8, 8, 8, 9, 10, 10

min

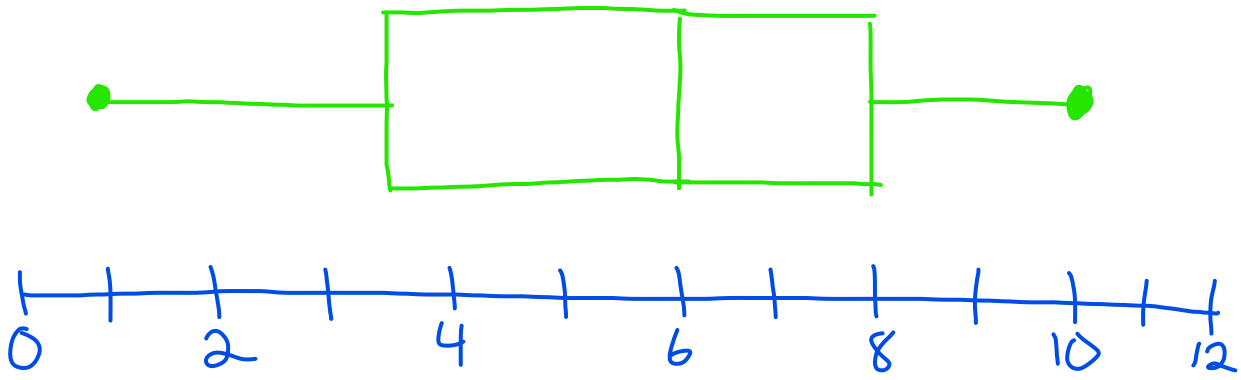
$\frac{3+4}{2}$ → $Q_1 = 3.5$

$\frac{6+6}{2}$ → $Q_2 = 6$

$\frac{8+8}{2}$ → $Q_3 = 8$

max

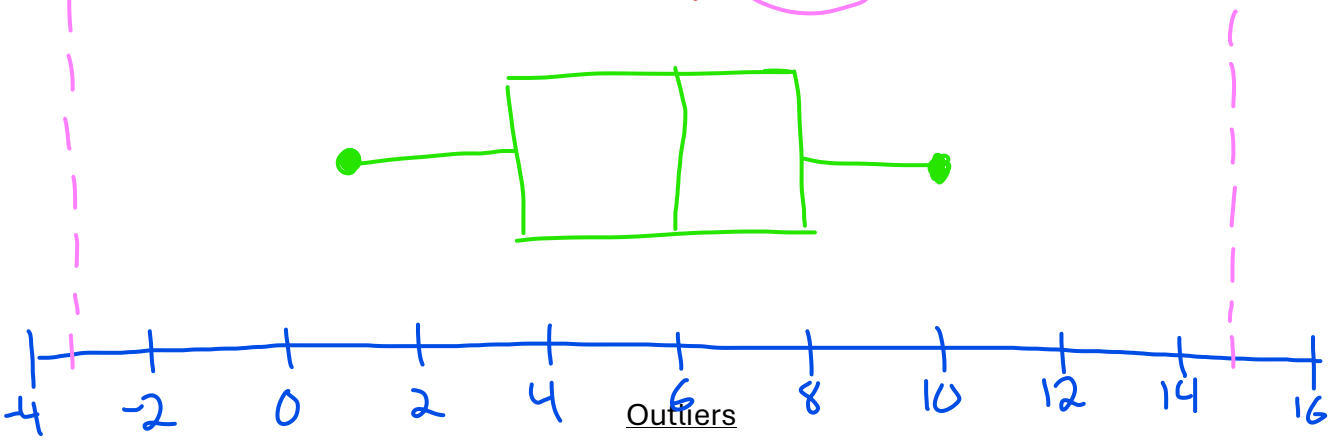
B. Using the five number summary you identified above, draw a box plot.



C. Still using the data given to you on question A, find the upper and lower fence, then draw a modified box plot. $IQR = Q_3 - Q_1 = 8 - 3.5 = 4.5$

$Upper = Q_1 - (1.5 \times IQR) = 3.5 - (1.5 \times 4.5) = -3.25$

$Lower = Q_3 + (1.5 \times IQR) = 8 + (1.5 \times 4.5) = 14.75$



Outliers

Knowing that the mean is 80 and the standard deviation is 7, determine if the values below are outliers. → $z\text{-score} = \frac{x-\mu}{\sigma}$ outlier when $z > |3|$

A. $\frac{60-80}{7} = -2.86$ No

B. $\frac{75-80}{7} = -0.71$ No

C. $\frac{43-80}{7} = -5.29$ Yes

D. $\frac{97-80}{7} = 2.43$ No

Probability Tables

1. A group of friends are playing a betting game based off of the sum of rolling a six-sided dice with a two-sided dice. Barabra wants to know the probabilities of each sum so she can bet with less risk. Create a probability table, then answer the questions below.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8

X	2	3	4	5	6	7	8
P	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

$X = 2, 3, 4, 5, 6, 7, 8$

(A) What is the probability of more than 5 but at most 7?

$P(5 < X \leq 7) = P(6) + P(7) = \frac{2}{12} + \frac{2}{12} = \frac{4}{12} = \frac{1}{3}$

wants 6 & 7

(B) What is the probability of at least 4?

$P(X \geq 4) = 1 - (P(2) + P(3)) = 1 - \left(\frac{1}{12} + \frac{2}{12}\right) = \frac{12}{12} - \frac{3}{12} = \frac{9}{12} = \frac{3}{4}$

wants 4–8

or add P(4) through P(8)

(C) What is the probability of less than 6?

$P(X < 6) = P(2) + P(3) + P(4) + P(5) = \frac{1}{12} + \frac{2}{12} + \frac{2}{12} + \frac{2}{12} = \frac{7}{12}$

wants 2–5

or $1 - (P(6) + P(7) + P(8))$

(D) What is the probability of 8?

$P(X = 8) = \frac{1}{12}$

(E) What is the probability of less than 4 or more than 5?

$P(X < 4 \text{ or } X > 5) = 1 - (P(4) + P(5)) = \frac{12}{12} - \left(\frac{2}{12} + \frac{2}{12}\right) = \frac{8}{12} = \frac{2}{3}$

wants 2, 3, 6, 7, & 8

or add P(2), P(3), P(6), P(7), & P(8)

(F) Find the mean, variance, and standard deviation of the probability table.

$\mu = \Sigma(x \cdot p) = \left(2 \cdot \frac{1}{12}\right) + \left(3 \cdot \frac{2}{12}\right) + \ldots + \left(8 \cdot \frac{1}{12}\right) = 5$

$\sigma^2 = \Sigma(x^2 \cdot p) - \mu^2 = \left[\left(2^2 \cdot \frac{1}{12}\right) + \ldots + \left(8^2 \cdot \frac{1}{12}\right)\right] - 5^2 = 3.17$

$\sigma = \sqrt{\sigma^2} = \sqrt{3.17} = 1.78$

Factorials and Combinations

Solve the following questions by hand on A and B, then by calculator on C and D.

A. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

B. ${}_8C_3 = \binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot 3 \cdot 2 \cdot 1} = 56$

C. $\frac{10!}{4!} = 151200$ via calc

D. ${}_{25}C_{18} = 480700$ via Calc

Binomial Random Variable

1. A high school athlete running sprints is recording the time it takes to complete each sprint to track her progress. Currently, her personal best is 15.53 seconds. During a training session, she runs a total of 10 sprints at varying times. Out of the 10, she is able to get 4 sprints within 30 seconds from her personal best. $n = 10$; $x = 4$

   a. What is the probability of success and failure?

   $p = \frac{x}{n} = \frac{4}{10} = 0.4$ $\quad q = 1 - p = 1 - 0.4 = 0.6$

   b. Write the probability function. (Hint: all functions must include the possible x-values)

   $p(x) = \binom{10}{x} 0.4^x \cdot .6^{10-x}$ $\quad x = 0, 1, 2, \ldots, 10$

   c. What is the probability of 5 successes for her next session? Find this via the probability function.

   $p(x=5) = \binom{10}{5} (0.4)^5 (0.6)^{10-5} = 0.20$

   d. What is the probability of 2 successes? Find this via calculator function. → binompdf

   $p(x=2) = \text{binompdf}(10, 0.4, 2)$

   $p(x=2) = 0.12$

e. What is the probability of at most 8 successes? → binomcdf [X≤#]

$P(X \le 8) = \text{binomcdf}(10, 0.4, 8) = 0.1198$

f. What is the probability of less than 4 successes or more than 9 successes?

$P(X < 4 \text{ or } X > 9) = P(X \le 3) + P(X = 10)$

$= \text{binomcdf}(10, 0.4, 3) + \text{binompdf}(10, 0.4, 10) = 0.38$

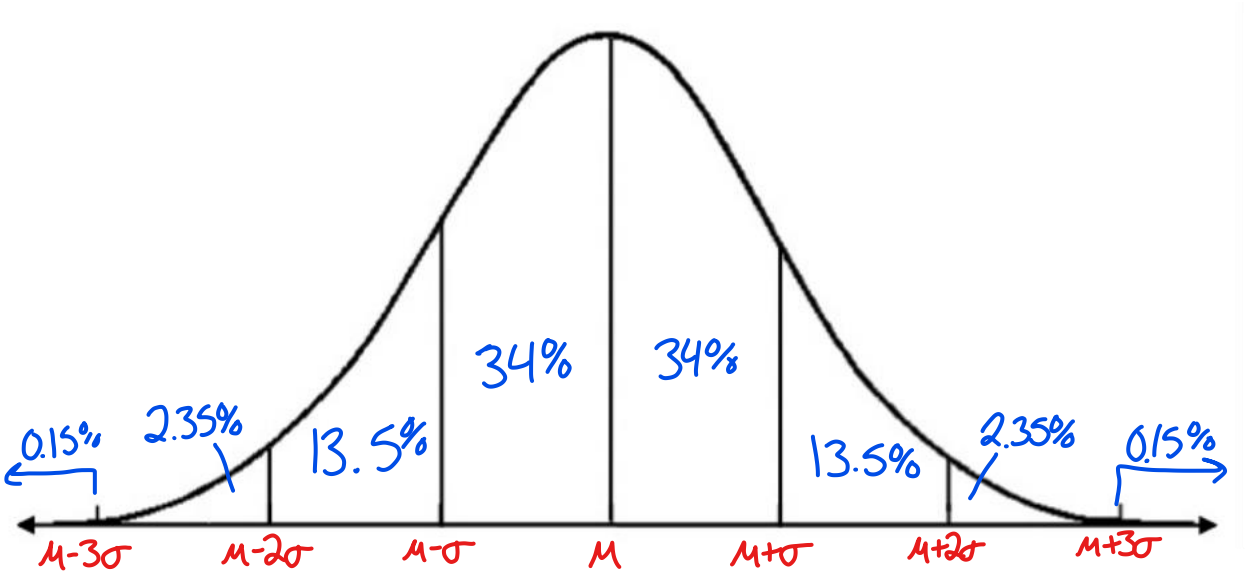
g. What is the probability of more than 3 but at ~~least~~ most 8 successes?

$P(3 < X \le 8) = P(X \le 8) - P(X \le 3)$

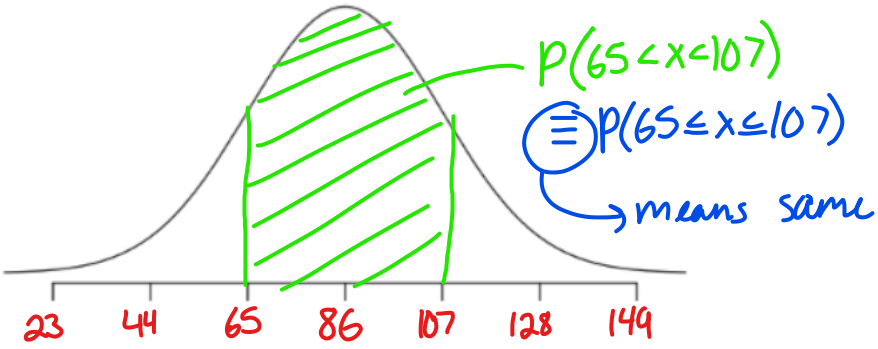
$= \text{binomcdf}(10, 0.4, 8) - \text{binomcdf}(10, 0.4, 3) = 0.62$

Chapter 5

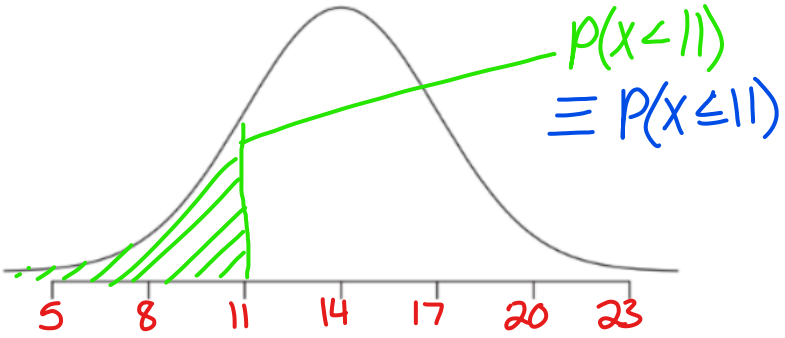
Empirical Rule



1. Sketch a graph in which the area (centered) is 68% while the mean is 86 and the standard deviation is 21. Write the Probability (P) statement for the shaded area.

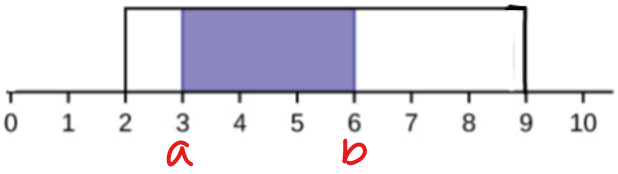


2. Sketch a graph in which the area (from the left) is 16% while the mean is 14 and the standard deviation is 3. Write the Probability (P) statement for the shaded area.



Uniform Distribution

1. Using the table below, answer the following questions.



a. What is the probability (i.e. area) of the shaded region?

$P(3<x<6) = \frac{1}{b-a} = \frac{1}{6-3} = \frac{1}{3}$

$\equiv P(3\leq x\leq 6)$

b. What would the probability of more than 2 but less than 7?

$P(2<x<7) = P(2\leq x\leq 7)$

$P(2\leq x\leq 7) = \frac{x_2 - x_1}{max - min} = \frac{7-2}{9-2} = \frac{5}{7}$

(9 = a, 2 = b)

c. What is the probability of exactly 5?

$P(x=5) = 0$

d. What would be the probability of less than 4 or at least 8?

$P(x<4 \text{ or } x\geq 8) = P(2\leq x\leq 4) + P(8\leq x\leq 9)$

$= \frac{4-2}{9-2} + \frac{9-8}{9-2}$

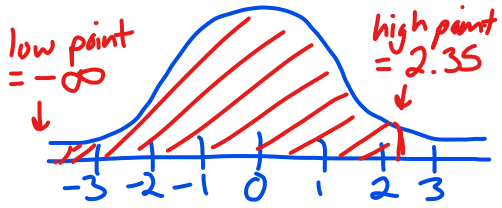
$= \frac{2}{7} + \frac{1}{7} = \frac{3}{7}$

Finding the Area

$N(0,1)$

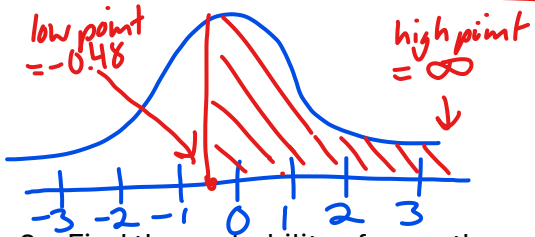
Solve the questions below. (Note: all will follow the standard normal distribution rule.)

1. Find the probability of less than 2.35.



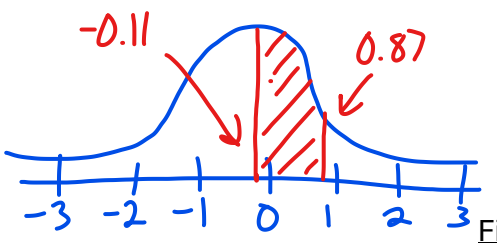
normalcdf(−E99, 2.35, 0, 1)
= 0.99

2. Find the probability of more than –0.48.



normalcdf(−0.48, E99, 0, 1)
= 0.68

3. Find the probability of more than –0.11 but less than 0.87.



normalcdf(−0.11, 0.87, 0, 1)
= 0.35

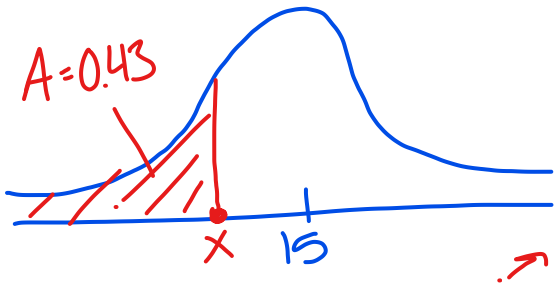
Finding the Point

$N(15, 7)$

Based off older calculator (where tail is always at LEFT)

Solve the questions below. (Note: all will follow the normal distribution where the mean is 15 and the standard deviation is 7.)

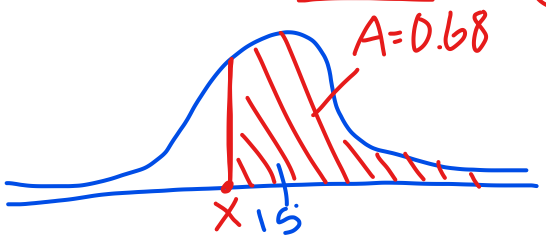
1. Find x when the area is 0.43 to the left.



invNorm(0.43, 15, 7, L)
= 13.77

2. Find x when the area is 0.68 to the right.

1 − 0.68 ≡ 0.32 to the ~~left~~



invNorm(0.32, 15, 7, L)
= 11.73

3. Find the x values when the area is 0.88 in the middle.

1 − 0.88 = 0.12

0.06 + 0.88 = 0.94

−X

invNorm(0.06, 15, 7, L)
= 4.12

+X

invNorm(0.94, 15, 7, L)
= 25.88