Goodness of Fit Practice

1. Sally wants to figure out which of the three highly recommended electric companies is the most recommended. To do so, she compares her survey ratings (shown below out of 150 people for each company) to the company's claimed satisfactory percentages: company A is 75%, company B is 60%, and company C is 80%. Determine if her rates line up with the company's claims with a 1% level of significance.

	Company A	Company B	Company C
Observed	79	80	68
Expected	112.5	90	120

a. State the Hypotheses.

H_0: $p_A = 0.75$; $p_B = 0.60$; $p_C = 0.80$ H_1: At least 1 is incorrect.

b. Calculate the Expected Counts. $E_i = n \cdot p_i$

$E_A = (150)(0.75) = 112.5$

$E_B = (150)(0.60) = 90$

$E_C = (150)(0.80) = 120$

c. What is the degrees of freedom?

$df = k - 1 = 3 - 1 = 2$

d. What is the Test Statistic? $TS = \sum \frac{(O-E)^2}{E}$

$$TS = \frac{(79-112.5)^2}{112.5} + \frac{(80-90)^2}{90} + \frac{(68-120)^2}{120} = 33.62$$

e. Calculate the p-value. What is the decision? $\alpha = 0.01$

$p = \chi^2cdf(33.62, E99, 2) = 5.01 \times 10^{-8} \approx 0.0000 < 0.01$

Reject H_0

f. Choose the correct interpretation.

- ☒ At 1% level of significance, there is sufficient evidence to support that the distribution proportions are not correct.
- ☐ At 1% level of significance, there is insufficient evidence to support that the distribution proportions are not correct.

2. In the previous years' track and field competition, out of the 60 students that participated from Coach Anderson's school, only 4 won gold, 8 won silver, 14 won bronze, and the remaining students got participation awards. Comparing the observed counts below, did Coach Anderson's school maintain it's previous years' achievements? [*Note: the level of significance is 10%.]

	Gold	Silver	Bronze	Participation
Observed	6	4	18	32
Expected	4	8	14	34

a. State the Hypotheses.

H_0: The distributions are correct. H_1: At least 1 is incorrect.

b. Calculate the Expected Counts. $E_i = n \cdot p_i$

$E_G = (60)\left(\frac{4}{60}\right) = 4$

$E_S = (60)\left(\frac{8}{60}\right) = 8$

$S_B = (60)\left(\frac{14}{60}\right) = 14$

$E_P = (60)\left(\frac{60-(4+8+14)}{60}\right) = 34$

c. What is the degrees of freedom?

$df = k - 1 = 4 - 1 = 3$

d. What is the Test Statistic?

$TS = \frac{(6-4)^2}{4} + \cdots + \frac{(32-34)^2}{34} = 4.26$

L_1 → observed & L_2 → expected then on $L_3 = (L_1 - L_2)^2 / L_2$

1-Var Stats → Σx

e. Calculate the p-value. What is the decision? $\alpha = 0.1$

$p = \chi^2cdf(4.26, E99, 3) = 0.2347 > 0.01$, Fail to Reject H_0

f. Choose the correct interpretation.

- ☐ At 10% level of significance, there is sufficient evidence to support that the distribution proportions are not correct.
- ☒ At 10% level of significance, there is insufficient evidence to support that the distribution proportions are not correct.

3. A bookstore manager wants to see whether the purchase rate of popular series declines over time or stays consistent. To do so, the manager looks at sells last year in April and sees that the purchase rate for Harry Potter was 38%, 33% for Percy Jackson, and 29% for Fourth Wing. When the manager observed sales for those series this April, it was seen that out of the 254 people that bought one of the three series, the proportions were as follows: 107 for Harry Potter, 77 for Percy Jackson, and 70 for Fourth Wing.

a. State the Hypotheses.

H_0: The proportions are correct. H_1: At least 1 is incorrect. → L_1

b. Calculate the Expected Counts.

$E_{HP} = (254)(0.38) = 96.52$

$E_{PJ} = (254)(0.33) = 83.82$ → L_2

$E_{FW} = (254)(0.29) = 73.66$

c. What is the degrees of freedom?

$df = k - 1 = 3 - 1 = 2$

d. Using the calculator trick, list the test statistic and p-value.

$TS = \chi^2 = 1.87$ $p = 0.3917$

e. What is your decision? $\alpha = 0.05$

$0.3917 > 0.05$, Fail to Reject H_0

f. Choose the correct interpretation.

- ❑ At 5% level of significance, there is sufficient evidence to support that the distribution proportions are not correct.
- ☒ At 5% level of significance, there is insufficient evidence to support that the distribution proportions are not correct.