Population Mean Confidence Interval Practice

1. A scientist conducting an experiment on reaction time for a new experiment design finds that the average reaction time is 2 hours with a standard deviation of 15 minutes. Note that the scientist has only conducted 15 trials due to budget constraints. n

   a. What type of distribution do you use and why?

   T-distribution b/c $n < 30$

   b. What is the point estimate?

   $PE = \bar{x} = 2\,hrs = \boxed{120\ min}$

   c. What is the margin of error? $t_{\alpha/2} = |invT(\frac{\alpha}{2}, df)|$

   $MOE = t_{\alpha/2}\frac{s}{\sqrt{n}} = \left|\cancel{invT(\tfrac{0.05}{2}}, 15-1)\right| \times \frac{15}{\sqrt{15}} = \boxed{8.29}$ (2.14)

   d. Construct a 95% confidence interval. $\alpha = 0.05$

   $(PE - MOE, PE + MOE)$

   $(120 - 8.29, 120 + 8.29) = \boxed{(111.71, 128.29)}$

   e. Interpret.

   We are 95% confident that the true unknown population mean lies in the interval (111.71, 128.29).

2. A basketball player is recording his progress throughout the off season on number of 3-point shots he can make. Assuming he attempts 75 shots per training session, he has an average of 35 successful shots with a standard deviation of 5 shots. n

   a. What type of distribution do you use and why?

   Z-distribution b/c $n \geq 30$

b. What is the point estimate?

$PE = \bar{x} = \boxed{35}$

c. What is the margin of error? $Z_{\alpha/2} = |\text{invNorm}(\frac{\alpha}{2}, \mu, \sigma)|$

$MOE = Z_{\alpha/2} \frac{s}{\sqrt{n}} = \left|\cancel{\text{invNorm}(\frac{0.1}{2}, 0, 1)}^{1.64}\right| \times \frac{5}{\sqrt{75}} = \boxed{0.95}$

d. Construct a 90% confidence interval. $\alpha = 0.1$

$(35 - 0.95, 35 + 0.95) = \boxed{(34.05, 35.95)}$

e. Interpret.

We are 90% confident that the true mean is between 34.05 and 35.95.

3. A professor finds that that the average test grade of his 48 students is 56 with a standard deviation of 8.

n

a. What type of distribution do you use and why?

Z-distributions b/c $n \geq 30$

b. What is the point estimate?

$PE = \bar{x} = \boxed{56}$

c. What is the margin of error?

$MOE = Z_{\alpha/2} \frac{s}{\sqrt{n}} = \left|\cancel{\text{invNorm}(\frac{0.01}{2}, 0, 1)}^{2.58}\right| \times \frac{8}{\sqrt{48}} = \boxed{2.98}$

d. Construct a 99% confidence interval. $\alpha = 0.01$

$$(56-2.98, 56+2.98) = \boxed{(53.02, 58.98)}$$

e. Interpret.

We are 99% sure that the true population mean is within the interval (53.02, 58.98).

4. A baker finds that on average, 14 batches of cookies are sold throughout a weekday, with a standard deviation of 0.75. Answer the questions below if the baker makes 23 batches.

n

a. What type of distribution do you use and why?

T-distribution b/c $n < 30$

b. What is the point estimate?

$$PE = \bar{x} = \boxed{14}$$

c. What is the margin of error?

$$MOE = t_{\alpha/2}\frac{s}{\sqrt{n}} = \left|\overset{2.07}{\cancel{\text{invT}\left(\frac{0.05}{2}, 23-1\right)}}\right| \times \frac{0.75}{\sqrt{23}} = \boxed{0.32}$$

d. Construct the confidence interval. Assume C.L. = 95% $\rightarrow \alpha = 0.05$

$$(14-0.32, 14+0.32) = \boxed{(13.68, 14.32)}$$

e. Interpret.

We are 95% confident that the true unknown population mean is between 13.68 and 14.32.