Population Confidence Interval Practice

1. Tracie is gathering data for a project via a survey sent out to the rest of her graduating class of 523. In the survey, she asks whether or not they have a job lined up for after graduation. Of the 523 people, 418 people said they have not found a job yet. Tracie has defined the success as the number of people who **have** found a job for after graduation. Solve the following questions with this information.

   n

   x

   523 − 418 = 105

   a. Is the sample large enough?

   $\hat{p} = \frac{105}{523} = 0.20$ $\quad \hat{q} = 1 - 0.20 = 0.80$

   $n\hat{p} \geq 10$ $\quad (523)(0.20) \geq 10$ $\quad = 104.6$ ✓

   $n\hat{q} \geq 10$ $\quad (523)(0.80) \geq 10$ $\quad = 418.4$ ✓

   Yes

   b. What is the point estimate?

   $PE = \hat{p} = 0.20$

   c. What is the margin of error? $Z_{\alpha/2} = |invNorm(\frac{\alpha}{2}, \mu, \sigma)|$

   $MOE = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = (|invNorm(\frac{.1}{2}, 0, 1)|)\sqrt{\frac{(0.2)(0.8)}{523}} = 0.03$

   1.64

   d. What is the 90% confidence interval?

   $(PE - MOE, PE + MOE)$

   $(0.20 - 0.03, 0.20 + 0.03) = (0.17, 0.23)$

   e. Is the confidence interval from the calculator the same as what you calculated?

   x = 105
   n = 523
   C-Level = 0.90

   Gives (0.17, 0.23) Yes

   f. Write the correct interpretation.

   We are 90% confident that the true population parameter lies in the interval (0.17, 0.23).

2. Suppose 150 people applied for a scholarship, but only 5 people were awarded. Estimate the proportion of applicants that are awarded a scholarship.

   n $\quad$ x

   a. Is the sample large enough?

   $\hat{p} = \frac{5}{150} = 0.03$ $\quad \hat{q} = 1 - 0.03 = 0.97$

$n\hat{p} \geq 10$ $(150)(0.03) \geq 10$ $= 4.5$ ✗ $\quad n\hat{q} \geq 10$ $(150)(0.97 \geq 18$ $= 145.5$ ✓ No → Wilson's

b. What is the point estimate?

$PE = \tilde{p} = \frac{x+2}{n+4} = \frac{5+2}{150+4} = \frac{7}{154} = 0.05$

c. What is the margin of error?

$MOE = z_{\alpha/2}\sqrt{\frac{\tilde{p}\tilde{q}}{n+4}} = (2.58)\sqrt{\frac{(0.05)(1-0.05)}{150+4}} = 0.05$

d. Construct a 99% confidence interval.

$(0.05-0.05, 0.05+0.05) = (0, 0.10)$

e. Is the confidence interval from the calculator the same as what you calculated?

$x = 7$
$n = 154$
$C.L = 0.99$
] Gives $(0.00, 0.09)$ No → rounding differs both answers slightly

f. Write the correct interpretation.

We are 99% confident that the true population proportion is in (0, 0.1).

3. Shelly saw that of the 314 people involved (n) in a survey about a restaurant's quality, 232 people (x) said they were satisfied with the quality. Solve the following questions with this information.

a. Is the sample large enough?

$\hat{p} = \frac{232}{314} = 0.74$ $\quad (314)(0.74) \geq 10$ $= 232.36$ ✓ $\quad (314)(0.26) \geq 10$ $= 81.64$ ✓ Yes

$\hat{q} = 1 - 0.74 = 0.26$

b. What is the point estimate?

$PE = \hat{p} = 0.74$

c. What is the margin of error?

$$MOE = \left(\left|\text{invNorm}\left(\frac{0.05}{2}, 0, 1\right)\right|\right)\sqrt{\frac{(0.74)(0.26)}{314}} = 0.05$$ (1.96)

d. Construct a 95% confidence interval.

$$(0.74 - 0.05, 0.74 + 0.05) = (0.69, 0.79)$$

e. Is the confidence interval from the calculator the same as what you calculated?

$x = 232$
$n = 314$
$C.L. = 0.95$
Gives $(0.69, 0.79)$ Yes

f. Write the correct interpretation.

We are 95% confident that the true population proportion is between 0.69 & 0.79.

4. Jonathan is testing a new recipe for lemon bars to possibly sell at his bakery. Before officially adding the dessert to his menu, he wants to see if it would sell well, so he makes the dessert a temporary menu option for 2 weeks. During those 2 weeks he saw that 1% of the 350 customers each week bought a lemon bar. Solve the following questions for the full 2 weeks. $350 \times 2 = 700 = n$ (p)

a. Is the sample large enough?

$\hat{p} = 0.01$ $(700)(0.01) \geq 10$
$\hat{q} = 0.99$ $= 7$ ✗ → No

Rounding to 4 decimals from here on!

b. What is the point estimate? $x = (700)(0.01) = 7$

$$PE = \tilde{p} = \frac{7+2}{700+4} = \frac{9}{704} = 0.0128$$

c. What is the margin of error?

$$MOE = \left(\left|\text{invNorm}\left(\frac{0.1}{2}, 0, 1\right)\right|\right)\sqrt{\frac{(0.0128)(1-0.0128)}{700+4}} = 0.0066$$ (1.5598)

d. Construct an 88% confidence interval.

$(0.0128-0.0066, 0.0128+0.0066) = (0.0062, 0.0194)$

e. Is the confidence interval from the calculator the same as what you calculated?

$X=9$
$n=704$
C.L. $=0.88$

Gives $(0.0062, 0.0194)$ Yes

f. Write the correct interpretation.

We are 88% confident that the true population parameter is in the interval $(0.0062, 0.0194)$.